Teacher notes
Topic E

The de Broglie hypothesis

We saw that de Broglie suggested in 1924 that to every material particle we should associate wave properties with a wavelength given by $\lambda = \frac{h}{p}$ where p is the momentum of the particle.

But where does this come from?

There is no way to derive this relation from principles of Physics which is why we still call it a "hypothesis". Its validity rests on the fact that it is verified in experiments such as the Davisson-Germer and Thomson experiments in which electrons exhibit wave behavior with a wavelength consistent with de Broglie's hypothesis.

So how did de Broglie come up with this hypothesis?

The answer comes from photons. A photon is a massless particle moving at the speed of light and so is described by special relativity. The energy of a particle of mass m and momentum p is given by Einstein's formula $E = \sqrt{(mc^2)^2 + p^2c^2}$ which for the massless photon becomes $E = pc$. But Einstein, in his explanation of the photelectric effect, argued that the energy of a photon is also given by $E = \frac{hc}{\lambda}$.

Putting the two together we get $pc = \frac{hc}{\lambda}$ which leads to the formula for the **momentum of the photon** to be

$$p = \frac{h}{\lambda}$$

This was confirmed by Compton in his scattering experiment of 1923 in which photons incident on electrons scattered with an increased wavelength. The experimental results were consistent with the photon behaving as an ordinary **particle with momentum**, the momentum given by the formula above.

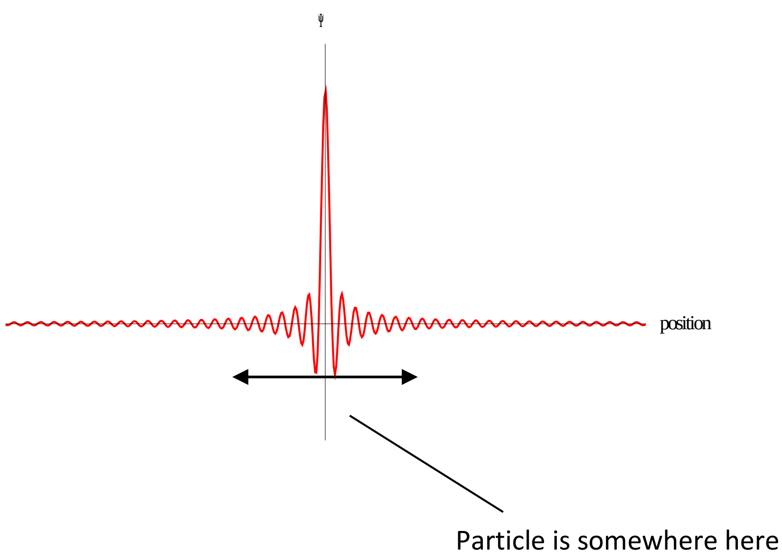
This is where de Broglie got the idea for his hypothesis. He turned the **photon** formula $p = \frac{h}{\lambda}$ around to write $\lambda = \frac{h}{p}$ for **particles** because he believed that what was true for photons should be true for material particles, like electrons, as well. So it is meaningless to ask whether the de Broglie formula applies to photons!

You will find "derivations" of the de Broglie formula which go like this:

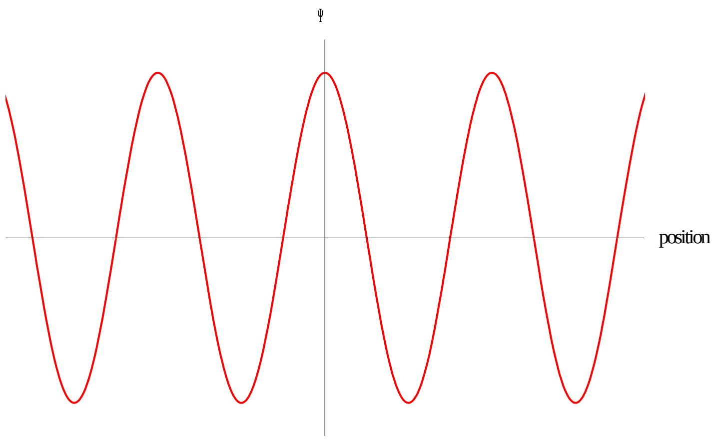
Einstein says that $E = mc^2$ for particles and $E = \frac{hc}{\lambda}$ for photons. Equating the two gives $mc^2 = \frac{hc}{\lambda}$ and so $\lambda = \frac{h}{mc}$. If we now pretend that we are dealing with a particle, we should replace *c* by the particle's speed *v* to get $\lambda = \frac{h}{mv}$ i.e. $\lambda = \frac{h}{p}$.

But this is a hand-waving argument that does not prove anything.

De Broglie describes a particle such as an electron with what is called a *wave packet*. A wave packet is a *superposition* of **many** sine waves whose wavelengths are centered around $\lambda = \frac{h}{p}$. In other words, the wave packet *does not just* contain the de Broglie wavelength. This superposition creates a wave that is localized in space and moves forward with a speed equal to the speed of the particle.



A wave made up of just one wavelength, the de Broglie wavelength, would look like:



This cannot represent a particle since the position of the particle is not localized. In addition, such a wave does not move at the speed of the particle it is supposed to represent.